REJOINDER.

The point at issue between Dr. Magee and myself concerns the reliability and accuracy of the second of the two methods which he uses to measure the correlation between two series of paired items. He says that the first method gives a "rough" measure of correlation, while the second is "a more accurate form of the degree of correspondenc." (QUARTERLY PUBLICATIONS, June, 1912, pp. 179–180). I agree with his characterization of the first method but hold that the second method not only gives erratic and unreliable results, but that it can not be considered a measure of correlation at all. Our difference is not merely a difference of opinion as to what constitutes perfect correlation.

Dr. Magee describes his "more accurate" form of the Degree of Correspondence as follows: "The percentages of increase or decrease for the corresponding items are computed, and the smaller is divided by the larger. After this number has been computed for each pair of values, the arithmetic mean is taken and the result gives the Degree of Correspondence in the form which considers the amount as well as the direction of the changes" (Money and Prices, p. 7).

Let us apply this method to the following hypothetical series:

YEAR.	A.	B.	C.	D.
1900	100.0	100.0	100.0	100.0
1901	101.0	110.0	101.0	110.0
1902	102.0	121.0	202.0	121.0
1903	103.0	133.1	181.8	119.8
1904	104.0	146.4	163.6	0

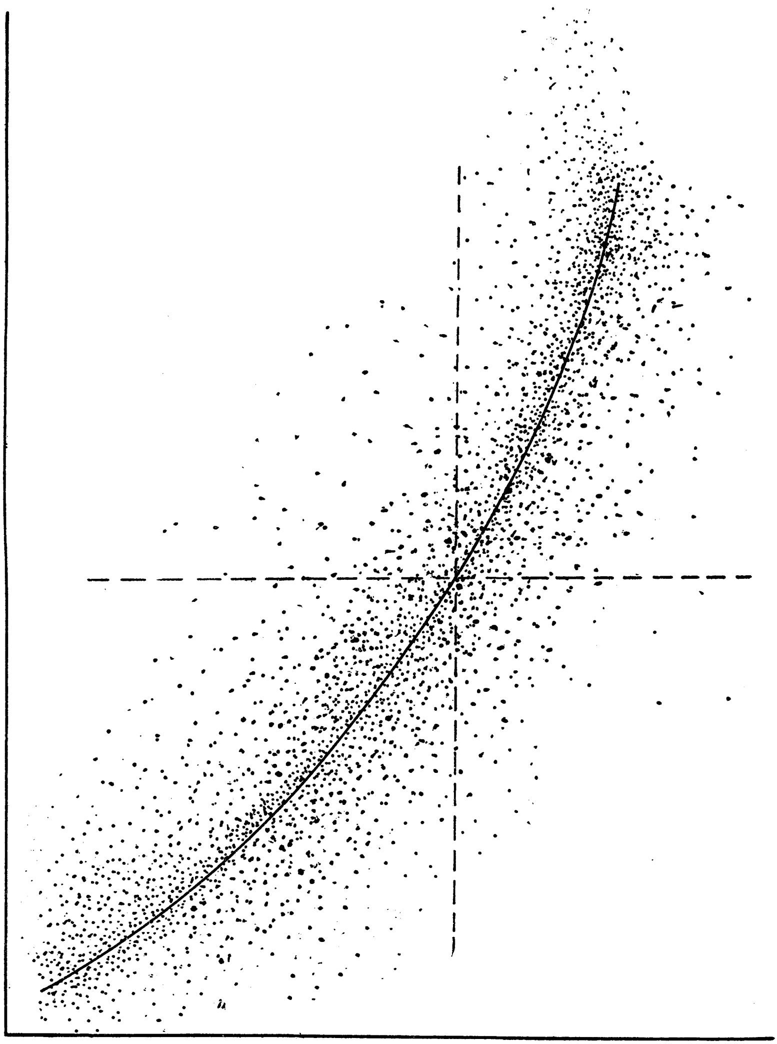
Columns A and B are increasing geometrical progressions, *i. e.*, each item is obtained from the preceding item by adding 1 per cent. and 10 per cent., respectively. The direction of the change is the same throughout the series A and B. The ratio of the percentages of increase is constant and equal to $\frac{1}{10}$, and the arithmetic average which gives the Degree of Correspondence is, therefore, $\frac{1}{10}$. If we should make the computation for series A and another geometrical progression with 2 per cent. as the rate of increase the Degree of Correspondence would be $\frac{1}{2}$. It is evident that two series which move together, each having a constant rate of change, may be built up to give any desired value to the Degree of Correspondence. It appears, then, that the coefficient measures not correlation but average ratio of the rates of change. However, interchanging the rates of change for corresponding items does not affect the result. To illustrate: Column C is constructed by adding 1 per cent., then 100 per cent., and subtracting 10 per cent., then 10 per cent.; column D is constructed by adding 10 per cent., then 10 per cent., and subtracting 1 per cent., then 100 per cent.

from or to each item to get the succeeding item. In each case the ratio of the percentages is $\frac{1}{10}$, making the Degree of Correspondence between series C and D equal to $\frac{1}{10}$, which was the result obtained for series A and B.

It is clear, then, that Dr. Magee uses the word "correlation" to indicate something quite different from the accepted connotation of that term. A. L. Bowley has well expressed the generally accepted meaning of correlation as follows: "When two quantities are so related that the fluctuations in one are in sympathy with the fluctuations of the other, so that an increase or decrease of one is found in connection with an increase or decrease (or inversely) of the other, and the greater the magnitude of the changes in the one, the greater the magnitude of the changes in the other, the quantities are said to be *correlated*" (Elements of Statistics, p. 316). Any coefficient of correlation, therefore, when applied to two time series should measure their *synchronization;* it should be large if fluctuations occur at the same time and if small or large fluctuations of one series occur with relatively small or large fluctuations of the other series. The Pearsonian Coefficient computed in the usual way for series A and B would be very high and positive, while for series C and D it would be low and negative.

The Degree of Correspondence does not give the extent of correlation, but it does give the average ratio of relative change, a distinctly different concept. If we let x represent any item in the A column, y the corresponding item in the B column, and n the number of years from 1900, then $x = 100 (1.01)^n$ and $y = 100 (1.10)^n$, giving $\frac{\log x - 2}{\log 1.01} = \frac{\log y - 2}{\log 1.10}$. The last equation states that for every increase of 1 per cent. in x there is an increase of 10 per cent. in y. It is to be noticed that this is a linear relation between log x and log y so that if the Pearsonian Coefficient of Correlation be computed for the logarithms of the numbers in columns A and B it will be $+1$, indicating the maximum degree of direct correlation. This, of course, results from the assumption that the corresponding items of the series A and B have an exact mathematical relationship existing between them, in this case the compound interest law. If we should use the corresponding values of columns A and B as coördinates the points thus located would be upon and determine a curve similar to the solid line in the accompanying diagram, the curve being the curve of regression. If, as would be the case when we are dealing with questions of correlation, the items of column A change *approximately* 1 per cent. while the corresponding items of column B change *approximately* 10 per cent., then the plotted points would be located in *proximity* to the curve. The Pearsonian Coefficient of Correlation, computed for the logarithms, gives a measure of the closeness of grouping of the points about the compound interest curve, the curve of regression in this case.

Dr. Magee has confused the idea of correlation (closeness with which the actual corresponding fluctuations obey any law which we may select) and the law which we select for the test. It happens that the law usually selected is the simplest, that is, the question we usually put is this: Does



an increase or decrease of a units in one series correspond to an increase or decrease (or inversely) of b units in the other series? In the case stated our curve of regression becomes a straight line and the Pearsonian Coefficient of Correlation measures the closeness of grouping of points about this straight line (see Yule's Theory of Statistics, p. 202).

Let me call attention to the fact that Dr. Magee's two methods give widely different results when applied to the same pair of series. The Degree of Correspondence for Kemmerer's figures for relative circulation and general prices comes out +0.48 and +0.20, respectively. When there is conflict Dr. Magee abides by the result of the second method, the one which I have criticized.

In conclusion it may be pointed out again that where yearly figures are correlated, such as the indices of prices of nine railway bonds on the New York Stock Exchange and the net deposits of New York Clearing House banks, like growth elements may be the cause of the high Degree of Correspondence obtained; the risk element in the bonds may be steadily decreasing as population increases along the railways and the net deposits may be steadily increasing because of the growth of New York banks due in turn to the growth of New York City.

WARREN M. PERSONS.